Heat Transfer (BTME- 501-18)

COURSE OUTCOMES	
CO1	Explain the basic principles of conduction, radiation, and convection heat transfer
CO2	Demonstrate an understanding of the concept of conservation of energy and its application to problems involving conduction, radiation, and/or convection heat transfer.
CO3	Solve engineering problems involving conduction heat transfer.
CO4	Identify, formulate, and solve engineering problems involving forced convection heat transfer, natural convection heat transfer, and heat exchangers.
CO5	Formulate and solve engineering problems involving radiation heat transfer among black surfaces and among diffuse gray surfaces.

UNIT 1

Introduction:
What, How, and Where?
Thermodynamics and Heat transfer
Application
Physical mechanism of heat transfer

Conduction: Introduction 1D, steady-state 2D, steady-state
Transient

Convection:

Introduction
External and internal flows Free convection
Boiling and condensation Heat exchangers

Radiation: Introduction View factors

Mass Transfer: Introduction Mass diffusion equation
Transient diffusion

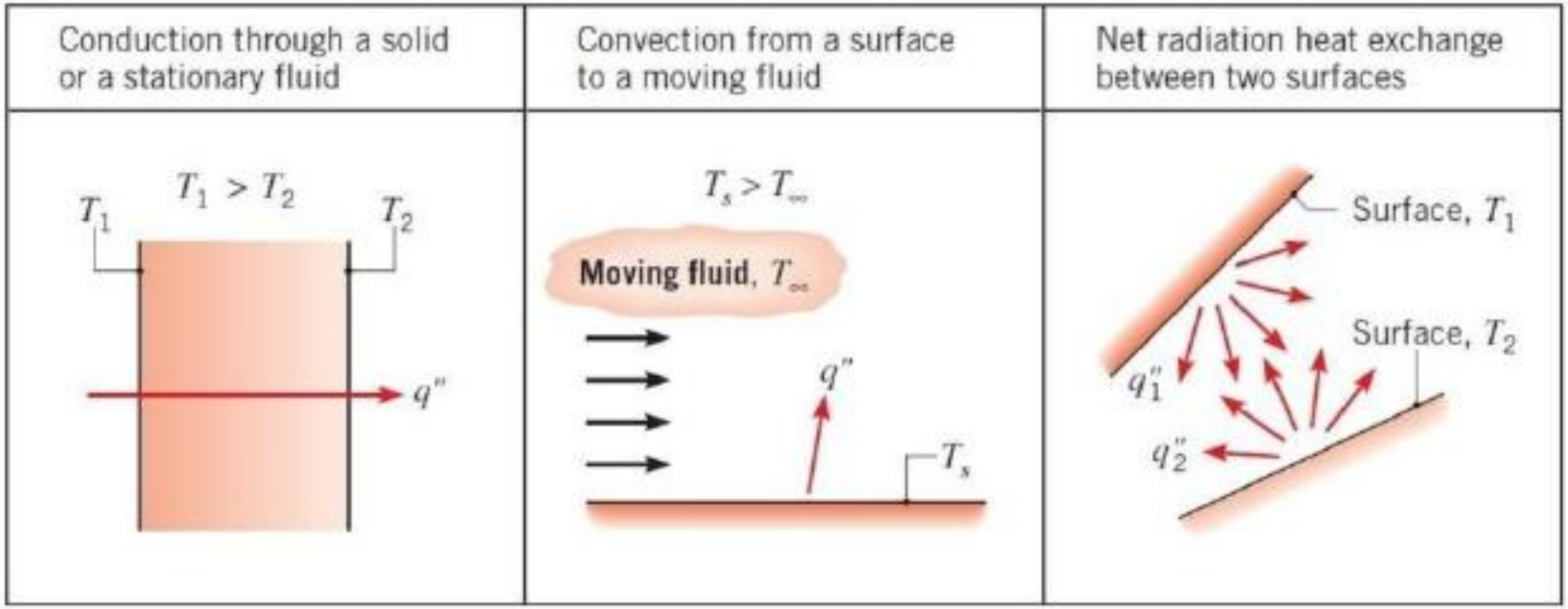
HEAT TRANSFER - WHAT?

- The science that deals with the determination of the rates of energy transfer due to temperature difference.

Driving force

- Temperature difference
  - as the voltage difference in electric current as the pressure difference in fluid flow
- Rate depends on magnitude of dT

Modes of Heat Transfer



Thermal Power Plant

Thermodynamics & Heat Transfer

Thermodynamics

Deals with the amount of energy (heat or work) during a process
Only considers the end states in equilibrium
Why?

Heat Transfer

Deals with the rate of energy transfer
Transient and non-equilibrium
How long?

Thermodynamics & Heat Transfer

Laws of Thermodynamics

Zeroth law - Temperature
First law Energy conserved
Secondlaw Entropy
Third law $S \rightarrow$ constant as $T \rightarrow 0$

Laws of Heat Transfer

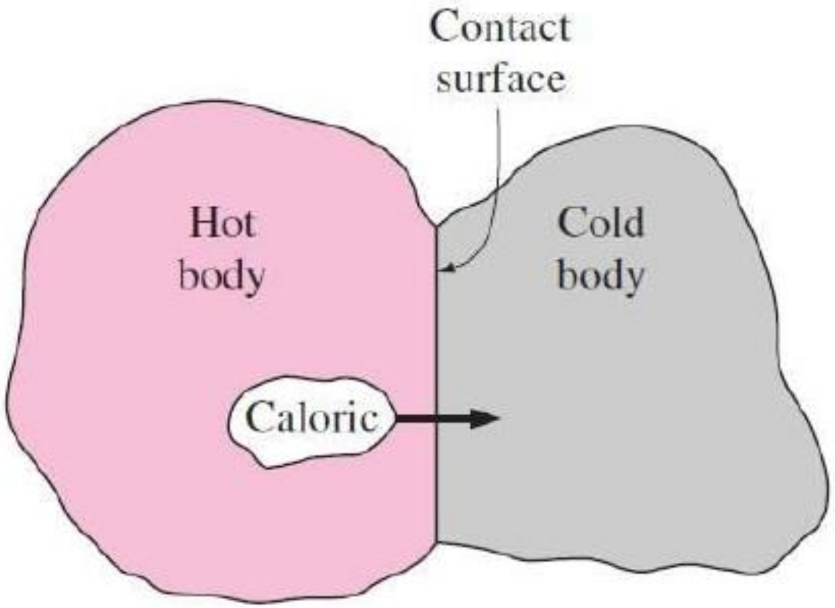
Fouriers law - Conduction
Newtons law of cooling - Convection
Stephan-Boltzmann law - Radiation

HISTORY OF HEAT TRANSFER

Caloric theory (18th Century)

Heat is a fluid like substance, '*caloric*' poured from one body into another.

Caloric: Massless, colorless, odorless, tasteless



Kinetic theory (19th Century)

Molecules - tiny balls - are in motion possessing kinetic energy

Heat: The energy associated with the random motion of atoms and molecules

HEAT, HEAT TRANSFER RATE, HEAT FLUX

Heat

The amount of heat transferred during a process, Q

Heat transfer rate

The amount of heat transferred per unit time, $\dot{Q}$ or simply q

$$Q = \int_0^{\Delta t} q\,dt$$

$$Q = q\Delta t, \text{ if } q \text{ is constant}$$

Heat flux

The rate of heat transfer per unit area normal to the direction of heat transfer:

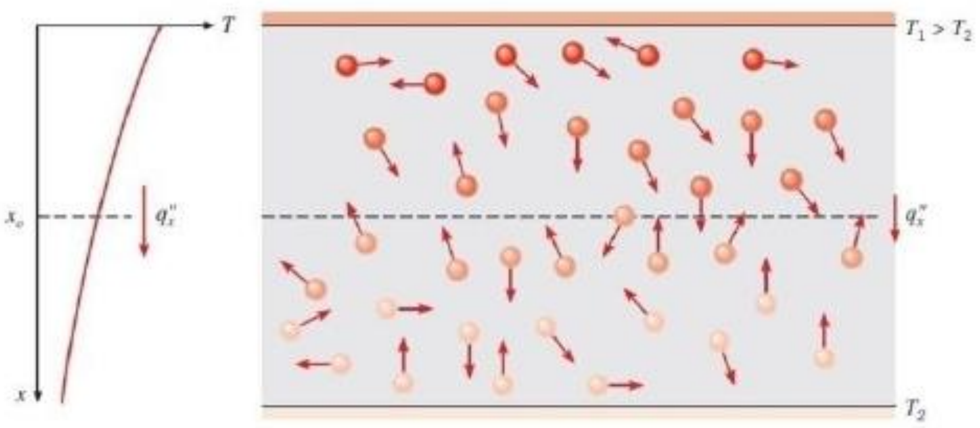
$$q'' = \frac{q}{A}$$

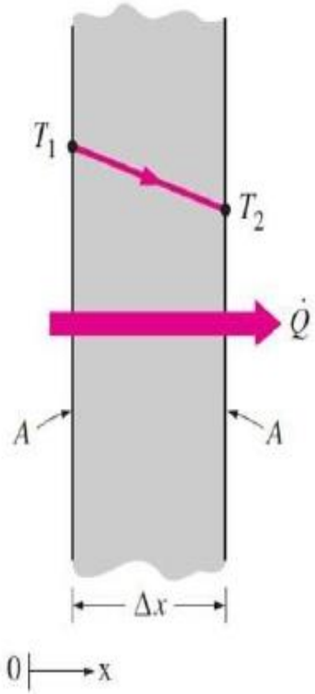
CONDUCTION

Viewed as

The transfer of energy from the more energetic to the less energetic particles of a substance due to interactions between the particles.

Net transfer by random molecules motion - *diffusion of energ*





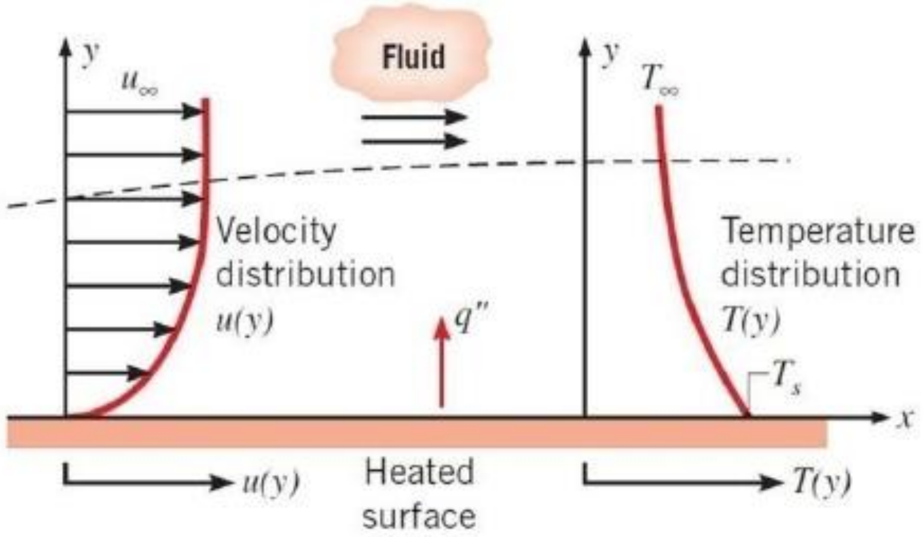
$$q_{cond} = -kA\frac{T_1 - T_2}{\Delta x} = -kA\frac{dT}{dx}$$

CONVECTION

Comprised of two mechanisms

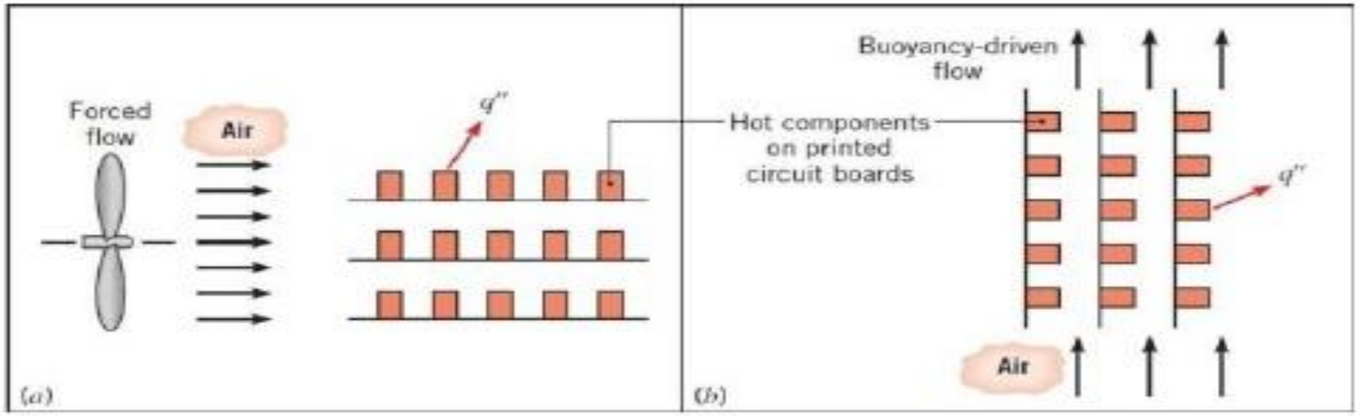
Energy transfer due to random molecular motion - *diffusion*

Energy transfer by the bulk motion of the fluid - *advection*

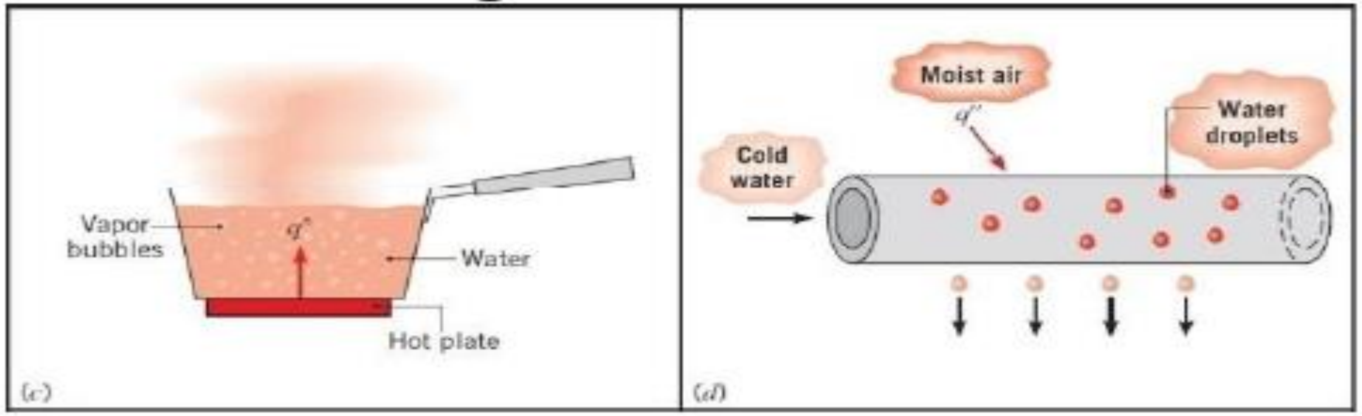


Boundary layer development in convection heat transfer

Forced and Free/Natural Convection



Boiling and Condensation



RADIATION

Radiation

Energy emitted by matter that is at a nonzero temperature

Transported by electromagnetic waves (or photons)

Medium?

Surface Emissive Power

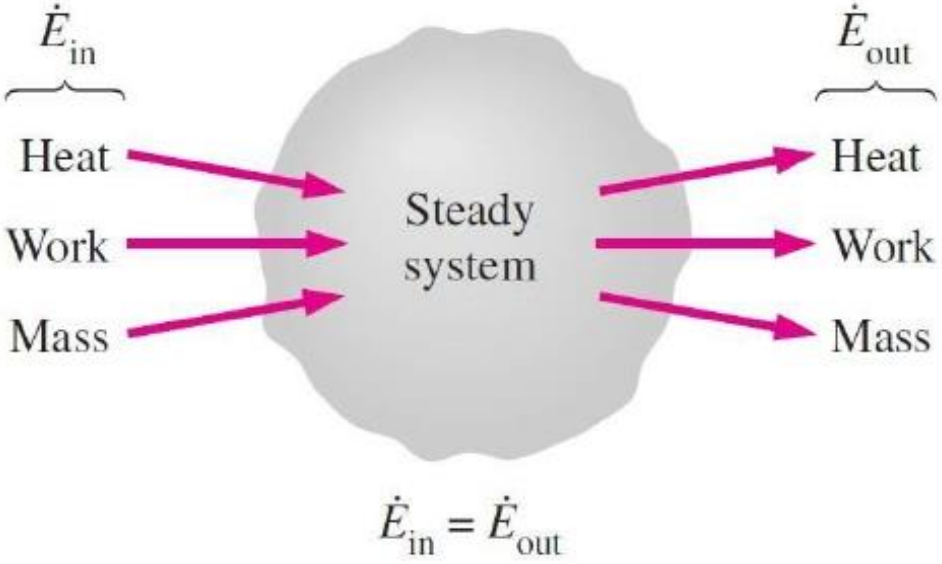
The rate at which energy is released per unit area (W/m^2)

$$E_b = \sigma T_s^4$$

$$q_{conv} = hA_s (T_s - T_\infty)$$

Process	h (W/m^2 K)
Free convection Gases Liquids	2-25 50-1000
Convection with phase change Boiling and Condensation	2500-100,000

Steady state with no heat generation

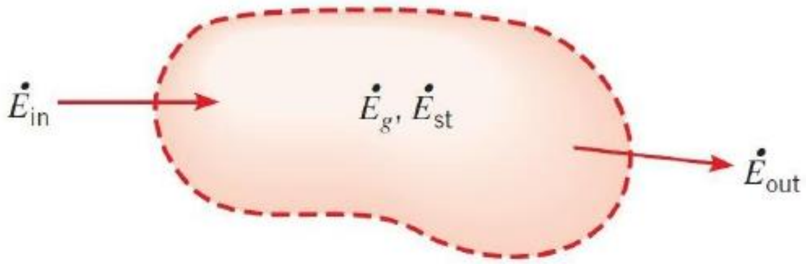


FIRST LAW OF THERMODYNAMICS

The inflow and outflow terms are *surface phenomena*. The *energy generation term* is a volumetric phenomenon.
  chemical, electrical
The *energy storage* is also a volumetric phenomenon.
  $\Delta U + \Delta KE + \Delta PE$
  ΔU : sensible/thermal, latent, and chemical components

$$E_{in} - E_{out} = \Delta E_{st}$$

In rate form:



$$\dot{E}_{in} - \dot{E}_{out} = \frac{dE_{st}}{dt} = \dot{E}_{st}$$

PROBLEM SOLVING: METHODOLOGY

Analysis of different problems will give a deeper appreciation for the fundamentals of the subject, and you will gain confidence in your ability to apply these fundamentals to the solution of engineering problems.

Be consistent in following these steps:

1. known
2. Find
3. Schematic
4. Assumptions
5. Properties
6. Analysis
7. Comments

STEADY STATE Vs. TRANSIENT

Fourier's law of heatconduction

$$q_{cond} = -kA\frac{dT}{dx}$$

transient
multidimensional - complex geometries

Steady-state heat transfer

- No change with time at any point within the medium
- T and q^{ij} remains unchanged with time
- $T = T(x, y, z)$
- Usually no but assumed

Transient heat transfer

- Time dependence
- $T = T(x, y, z, t)$
- Special case - *lumped* - T changes with time but not with location:

  $T = T(t)$

HEAT FLUX DIRECTION

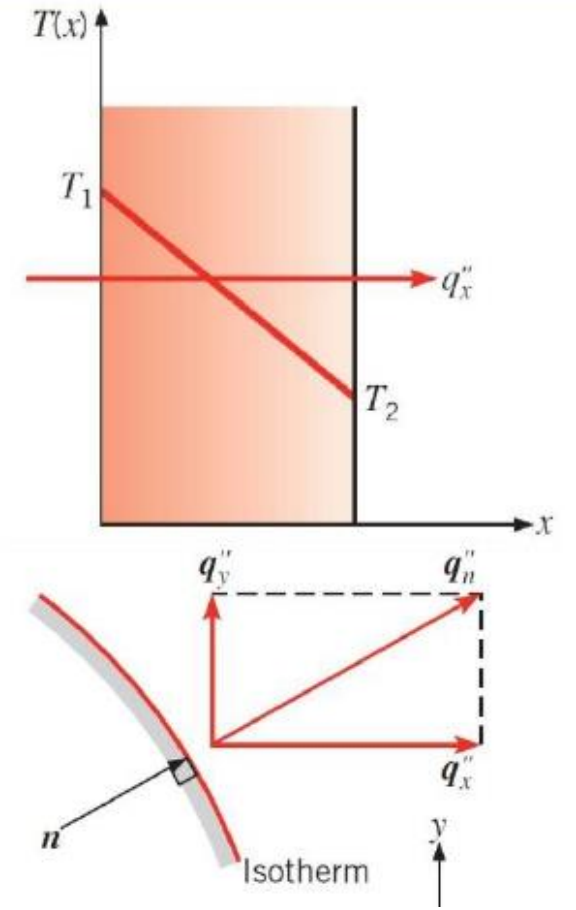
The direction of heat flow will always be normal to a surface of constant temperature, called an isothermal surface.

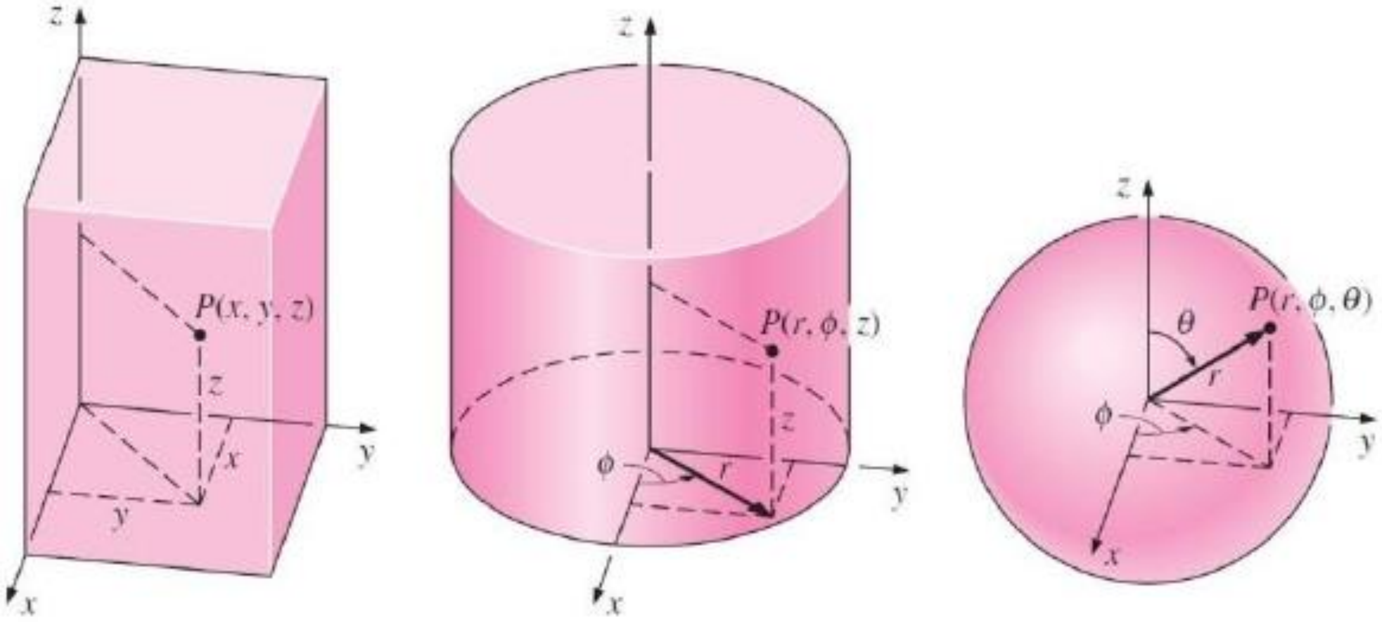
$$q''_x = -k\frac{\partial T}{\partial x};\; q''_y = -k\frac{\partial T}{\partial y};\; q''_z = -k\frac{\partial T}{\partial z}$$

$$\begin{aligned} q''_n &= q''_x i + q''_y j + q''_z k \\ &= -k\left(\frac{\partial T}{\partial x} i + \frac{\partial T}{\partial y} j + \frac{\partial T}{\partial z} k\right) \\ &= -k\nabla T \end{aligned}$$

where n is the normal of the isothermal surface and

$$q''_n = -k\frac{\partial T}{\partial n}$$





(a) Rectangular coordinates (b) Cylindrical coordinates (c) Spherical coordinates

THERMAL CONDUCTIVITY

Thermal conductivity

$$k = \frac{q''}{(\partial T/\partial x)}$$

The rate of heat transfer through a unit thickness of the material per unit area per unit temperature difference.

Specific heat, C_p

Ability to store thermal energy.
At room temperature,

$$C_p = 4.18 \text{ kJ/kg K, water}$$
$$= 0.45 \text{ kJ/kg K, iron}$$

Thermal conductivity, k

Material's ability to conduct heat
At room temperature,

$$k = 0.607 \text{ W/m K, water}$$
$$= 80.2 \text{ W/m K, iron}$$

- Transport property
- Indication of the rate at which energy is transferred by the diffusion process
- Depends on the physical structure of matter, atomic and molecular, related to the state of the matter
- *Isotropic* material - k is independent of the direction of transfer, $k_x = k_y = k_z$

  Laminated composite materials and wood

  k across grain is different than that parallel to grain

K For Different Materials at T and P

Kinetic theory of gases:

$$v_{rms} = \sqrt{\frac{3RT}{M}}$$

$T\uparrow \quad k\uparrow$

$M\uparrow \quad k\downarrow$

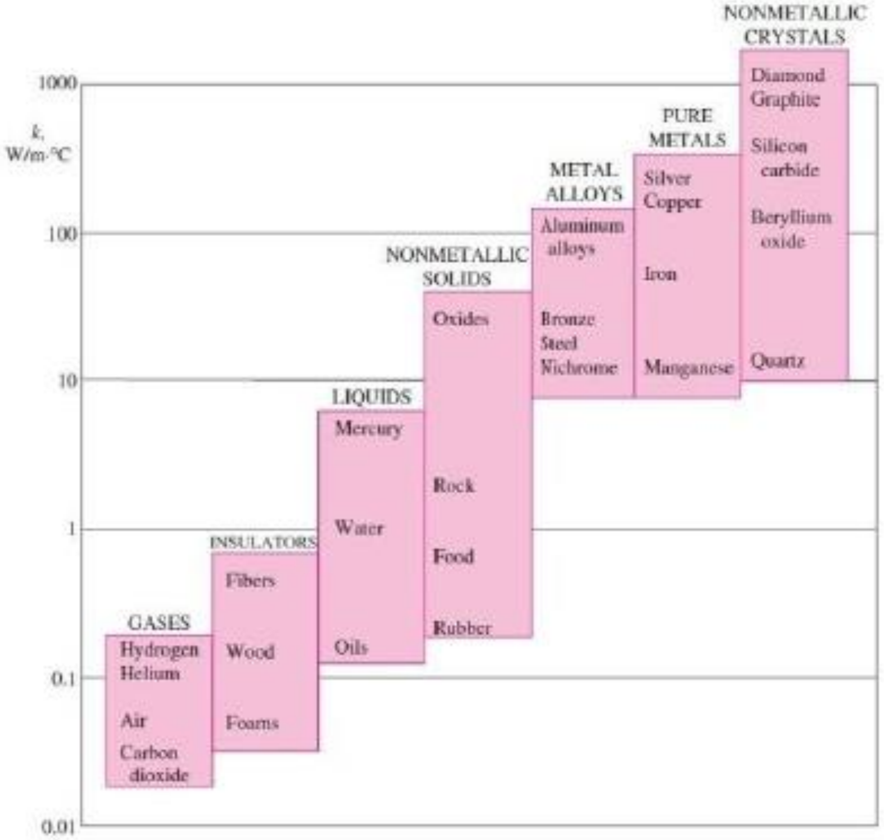
He(4), Air(29)

Liquids: Strong intermolecular forces

Most liquids: $T\uparrow \quad k\downarrow$

$M\uparrow \quad k\downarrow$

Except water: Not a linear trend



THERMAL DIFFUSIVITY

- Thermophysical properties
  - k Transport property
  - $\rho,\ C_p$ Thermodynamic properties
- ρC_p is volumetric heat capacity (J/m³ K)

- High α: faster propagation of heat into the medium
- Small α: heat is mostly absorbed by the material and a small amount of heat is conducted further

Heat Diffusion Equation

Governing Equation

$$\dot{E}_{in} - \dot{E}_{out} + \dot{E}_{g} = \dot{E}_{st}$$

$$q_x + q_y + q_z - q_{x+dx} - q_{y+dy} - d_{z+dz} + \dot{e}_g dxdydz = \rho C_p \frac{\partial T}{\partial t} dxdydz$$

$$-\frac{\partial q_x}{\partial x} dx - \frac{\partial q_y}{\partial y} dy - \frac{\partial q_z}{\partial z} dz + \dot{e}_g dxdydz = \rho C_p \frac{\partial T}{\partial t} dxdydz$$

However,

$$q_x = -k dydz \frac{\partial T}{\partial x}; \quad q_y = -k dxdz \frac{\partial T}{\partial y}; \quad q_z = -k dxdy \frac{\partial T}{\partial z}$$

$$\frac{\partial}{\partial x}\left(k \frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(k \frac{\partial T}{\partial y}\right) + \frac{\partial}{\partial z}\left(k \frac{\partial T}{\partial z}\right) + \dot{e}_g = \rho C_p \frac{\partial T}{\partial t}$$

Cases

Fourier-Biot equation - Isotropic

$$\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial x^2}+\frac{\dot{e}_g}{k}=\frac{1}{a}\frac{\partial T}{\partial t}$$

Diffusion equation - Transient, no heat generation

$$\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial x^2}=\frac{1}{a}\frac{\partial T}{\partial t}$$

Poisson equation - Steady-state

$$\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial x^2}+\frac{\dot{e}_g}{k}=0$$

Laplace equation - Steady-state, no heat generation

$$\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial x^2}=0$$

Coordinate Systems

Cartesian coordinates $T(x, y, z)$

$$\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right) + \frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right) + \dot{e}_g = \rho C_p \frac{\partial T}{\partial t}$$

Cylindrical coordinates $T(r, \varphi, z)$

$$\frac{1}{r}\frac{\partial}{\partial r}\left(kr\frac{\partial T}{\partial r}\right) + \frac{1}{r^2}\frac{\partial}{\partial \varphi}\left(k\frac{\partial T}{\partial \varphi}\right) + \frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right) + \dot{e}_g = \rho C_p \frac{\partial T}{\partial t}$$

Spherical coordinates $T(r, \varphi, \theta)$

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(kr^2\frac{\partial T}{\partial r}\right) + \frac{1}{r^2 \sin^2\theta}\frac{\partial}{\partial \varphi}\left(k\frac{\partial T}{\partial \varphi}\right) + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial \theta}\left(k\sin\theta\frac{\partial T}{\partial \theta}\right) + \dot{e}_g = \rho C_p \frac{\partial T}{\partial t}$$

Boundary and Initial Conditions

- Necessary to solve the appropriate form of the heat equation
  - Depends on the physical conditions at boundaries
  - On time
- Boundary conditions can be simply expressed in mathematical form
  - Second order in space, two boundary conditions for each coordinate needed to describe the system
  - First order in time, only one condition, *initial condition* must be specified

One-Dimensional, Steady -State

- Temp. gradients exist along only a single coordinate direction
- Heat transfer occurs exclusively in that direction
- Temp. at each point is independent of time

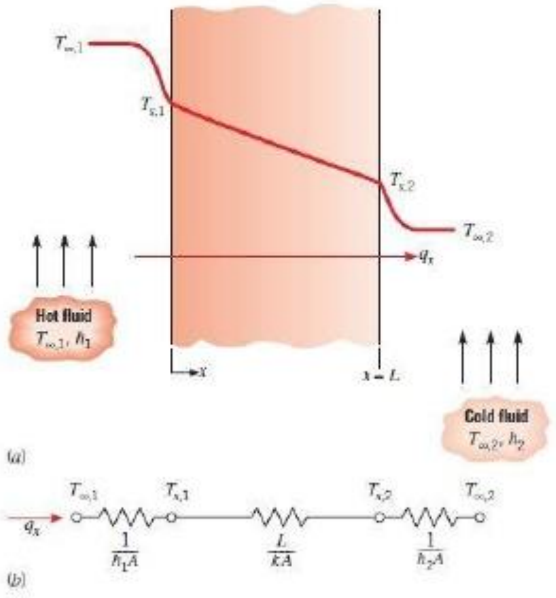
We will see:

- Temp. distribution & heat transfer rate in common (planar, cylindrical and spherical) geometries
- Thermal resistance
  - Thermal circuits to model heat flow
  - Electrical circuits to current flow

Cartesian Coordinates: T (x)

$$\frac{d}{dx}\left(k\frac{dT}{dx}\right) = 0$$

For 1-D, steady-state conduction in a plane wall with no heat generation, heat flux is a constant, independent of x.



Plane Wall

If k is constant then, $T(x) = C_1 x + C_2$

$$T(0) = T_{s,1} \quad \text{and} \quad T(L) = T_{s,2}$$

$$T(x) = (T_{s,2} - T_{s,1})\frac{x}{L} + T_{s,1}$$

$$q_x = -kA\frac{dT}{dx} = \frac{kA}{L}(T_{s,2} - T_{s,1})$$

$$q''_x = \frac{k}{L}(T_{s,2} - T_{s,1})$$

Thermal Resistance

Ratio of driving potential to the corresponding transfer rate

$$R_{t,cond} = \frac{(T_{s,1} - T_{s,2})}{q_x} = \frac{L}{kA}$$

$$R_e = \frac{E_{s,1} - E_{s,2}}{I}$$

$$R_{t,conv} = \frac{(T_s - T_\infty)}{q} = \frac{1}{hA}$$

Under steady state conditions:

$$\left|\text{Convection rate into the wall}\right| = \left|\text{Conduction rate through the wall}\right| = \left|\text{Convection rate from the wall}\right|$$

$$q_x = \frac{T_{\infty,1} - T_{s,1}}{1/h_1 A} = \frac{T_{s,1} - T_{s,2}}{L/kA} = \frac{T_{s,2} - T_{\infty,2}}{1/h_2 A}$$

$$q_x = \frac{T_{\infty,1} - T_{\infty,2}}{R_{tot}} \qquad R_{tot} = \frac{1}{h_1 A} + \frac{L}{kA} + \frac{1}{h_2 A}$$

Thermal Resistance: Radiation

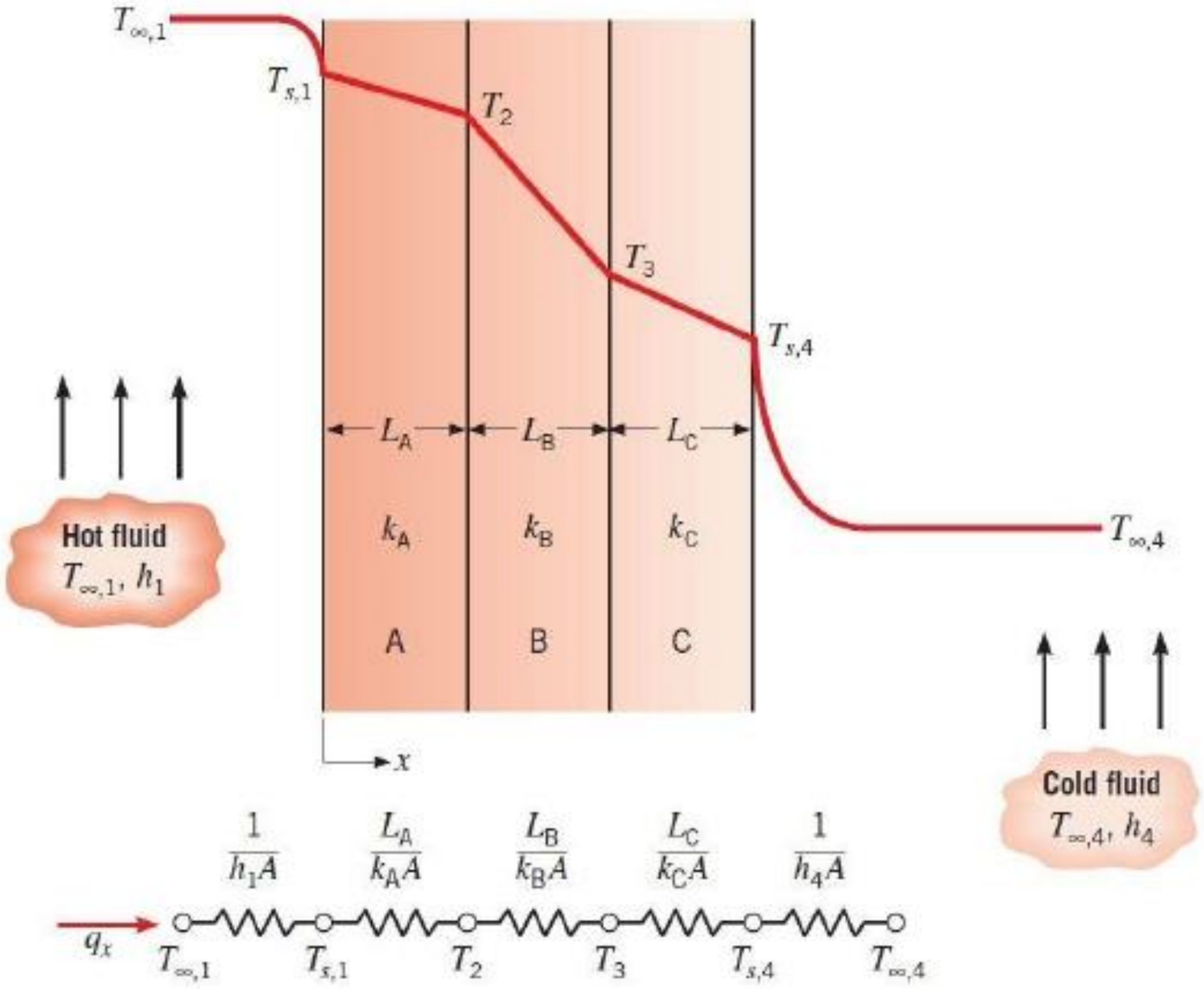
The **thermal resistance for radiation** - radiation exchange between the surface and its surroundings:

$$R_{t,rad} = \frac{T_s - T_{sur}}{q_{rad}} = \frac{1}{h_r A}$$

$$q_{rad} = h_r A (T_s - T_{sur})$$

The radiation heat transfer coefficient, h_r:

$$h_r = \varepsilon\sigma (T_s + T_{sur}) (T_s^2 + T_{sur}^2)$$


T∞,1
Ts,1
T2
T3
Ts,4
T∞,4
LA LB LC
kA kB kC
A B C
Hot fluid
T∞,1, h1
Cold fluid
T∞,4, h4
x
1/h1A LA/kAA LB/kBA LC/kCA 1/h4A
qx
T∞,1 Ts,1 T2 T3 Ts,4 T∞,4

Cylinder

The governing equation for 1D, steady state conduction in cylindrical coordinates:

$$\frac{1}{r}\frac{d}{dr}\left(kr\frac{dT}{dr}\right) = 0$$

The heat flux by Fourier's law of conduction,

$$q_r = -kA\frac{dT}{dr} = -k(2\pi rL)\frac{dT}{dr}$$

- Here, $A = 2\pi rL$ is the area normal to the direction of heat transfer.
- The quantity $\frac{d}{dr}\left(kr\frac{dT}{dr}\right)$ is independent of r
- The conduction heat transfer rate q_r (not the heat flux, q_r'') is a constant in the radial direction

Temperature distribution and heat transfer rate

$$T(r) = \frac{T_{s,1} - T_{s,2}}{\ln(r_1/r_2)} \ln\left(\frac{r}{r_2}\right) + T_{s,2}$$

Note that the temperature distribution associated with radial conduction through a cylindrical wall is logarithmic, not linear, as it is for the plane wall.

$$q_r = \frac{2\pi Lk\,(T_{s,1} - T_{s,2})}{\ln(r_2/r_1)}$$

Note that q_r is independent of r.

$$R_{t,cond} = \frac{\ln(r_2/r_1)}{2\pi Lk}$$